Determination of Saving Potential for a Parallel Hybrid Power Train (using Forward-looking Information)

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Abstract

Anticipatory driving can lower fuel consumption of a vehicle. Because of the ability to store energy, anticipation is even more promising for hybrid power trains. The knowledge of the road to come (by means of digital road-maps) allows an energy optimal operation of the hybrid power train.

An energy based investigation is not sufficient for potential-determination, because the degrees of freedom are limited by constraints. For a realistic determination of saving potentials, degrees of efficiency, storage space and maximum power of the components have to be considered. The potential of a forward-looking strategy of operation results from an optimal progression of control variables over the driving route. Control variables of a parallel hybrid power train are driving torques of combustion and electrical engine and chosen gear ratio. The determined energy consumption over the given route and given speed progression can be used as a benchmark for an implemented real-time strategy of operation.

A concept for determination of optimal progression of control variables and thus a way to determine optimal energy consumption is introduced within this paper. The concept is demonstrated by a parallel-hybrid vehicle featuring a natural-gas combustion engine.

1 Introduction

Speaking about parallel-hybrid drive trains is sometimes difficult, because using the same vocabulary as for regular drive trains, consisting of combustion engine followed by one clutch and a gear-box, does not always apply to parallel-hybrid drive trains. The parallel-hybrid drive train examined in this paper (see **Fig 1-1**) consists of a combustion engine followed by a clutch, connecting it to an electrical engine, followed by a second clutch, connecting the engines to the gearbox. In this paper the clutch between combustion and electrical engine is referred to as clutch1. The clutch between the electrical engine and the gearbox is referred to as clutch2. The combination of combustion and electrical engine is referred to as the engine throughout this paper, as it replaces the combustion engine of a regular drive train.

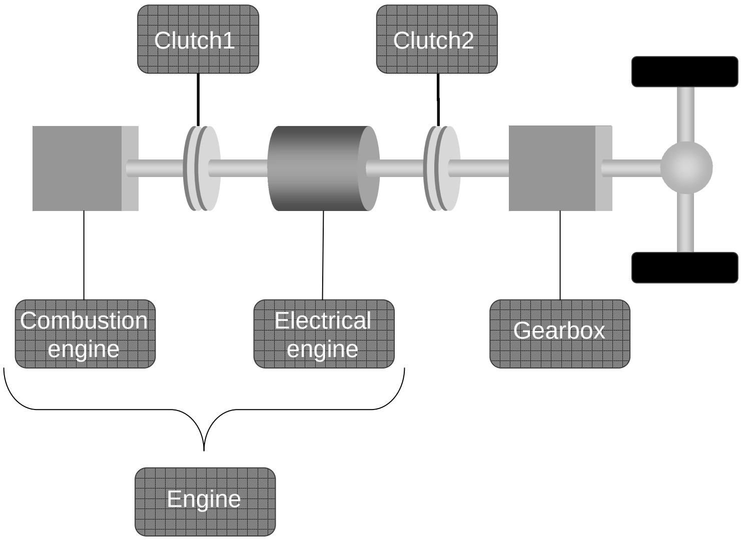

Fig 1-1 Power train structure

To be able to determine fuel consumption (and thus CO_2 exhaust) over a given route, the mechanical energy needed to drive the vehicle needs to be known. To determine the progression of the driving resistance, the elevation and speed progressions as well as certain parameters of the vehicle such as weight, wind drag coefficient, cross-sectional area and rolling resistance coefficient have to be known. From the progression of wheel speed and driving torque at the driving wheels the needed engine torque and engine speed for each gear can be calculated by a simple vehicle model. This is referred to as „backward simulation“ as the engine torque and speed are results of the vehicle speed, reversing the causal chain.

Knowing the engine speed and torque needed throughout the route for each gear, the optimal way to meet these requirements (i.e. the required driving power) has to be determined. This is done by optimization methods resulting in vectors of gear ratio and electrical engine torque.

2 Vehicle model

To determine the required driving power throughout the driving route, a mathematical vehicle model has to be developed. As only energy consumption related issues are regarded, influences by elasticity and play in the drivetrain can be disregarded. Thus the influences to be modelled are driving powers, degrees of efficiency of the electrical system and the CO_2 emissions of the combustion engine.

Driving powers

The required driving power is dependent on the driving resistance at each time over the driving course. The driving resistance is the sum of the forces counteracting the vehicle movement (except the slope resistance that can support vehicle movement in case of downward slope):

$$F_{dr} = F_{rr} + F_{wd} + F_{a} + F_{sl} \tag{2.1}$$

F_{dr} … driving resistance
F_{rr} … rolling drag
F_{wd} … wind drag
F_{a} … acceleration resistance
F_{sl} … slope resistance

The driving resistance consists of the wind drag, rolling drag, acceleration resistance and slope resistance. These result in the driving resistance at the driving wheels. The driving resistance force has to be multiplied by the wheel diameter to determine the driving torque at the driving wheel. In addition to the wheel driving torque component resulting from the driving resistance, an additional torque component resulting from angular acceleration of the wheels has to be added. Although only 2 wheels are driven, obviously all four wheels have to be accelerated as the car drives on the road. So the torque of inertia of all 4 wheels has to be considered.

$$\tau_{wh} = F_{dr} \times r_{wh} + 4J_{wh} \times \dot{\omega}_{wh} \tag{2.2}$$

τ_{wh} … wheel driving torque
F_{dr} … driving resistance (3.1)
r_{wh} … wheel diameter
J_{wh} … moment of inertia of the wheel
$\dot{\omega}_{wh}$ … angular acceleration

As only quasi-static driving states are regarded in this work, clutch slip is neglected. Thus the rotational speed relation between wheels and engine always equals the gear ratio. To determine driving torque at gearbox entry, the wheel driving torque has to be divided by the product of axle and gear ratio. Additionally the drive train efficiency has to be regarded. Although the efficiency of a drive train has constant, speed and load dependent losses [2] , a constant relation between output and input power is appropriate for this kind of research.

$$P_{out} = P_{in} \times \eta \Rightarrow \omega_{out} \times \tau_{out} = \omega_{in} \times \tau_{in} \times \eta$$

$$\tau_{in} = \frac{\omega_{out} \times \tau_{out}}{\omega_{in} \times \eta} = \frac{\tau_{out}}{i_n \times \eta} \tag{2.3}$$

$P_{in,out}$ … in/output powers

$\tau_{in.out}$ … in/output torques

$\omega_{in.out}$ … in/output angular velocity

i_n … overall gear ratio in gear n

η … drive train degree of efficiency

As the diameters of drive-shafts and gearbox-shafts are low in comparison to wheel and engine diameters, their moment of inertia will be neglected in this research. Their moment of inertia could be added to the wheels or the engine, multiplied by the square of the ratio [1] As the consumption maps for the engines are measured in static operating points, their moments of inertia have to be accounted for in addition to the gearbox entry torque. As the required torques are calculated before the operation mode is chosen, it is not known if the combustion engine has to be accelerated or not (e.g. if clutch1 is open or closed). Therefore the moment of inertia of the combustion engine is always added, assuming it is always revolving.

$$\tau_{en} = \frac{\tau_{wh}}{i_n \times \eta} + \dot{\omega}_{wh} \times i_n \times J_{en} \tag{2.4}$$

τ_{en} … required engine torque

J_{en} … moment of inertia of the engine (engines + clutches)

$$\omega_{en} = \omega_{wh} \times i_n \tag{2.5}$$

ω_{en} … engine angular velocity

With 3.4 & 3.5 the required engine torque in all gears for a given course can be calculated**. Fig 2-1** shows the required engine torques in all the gears compared to the vehicle speed.

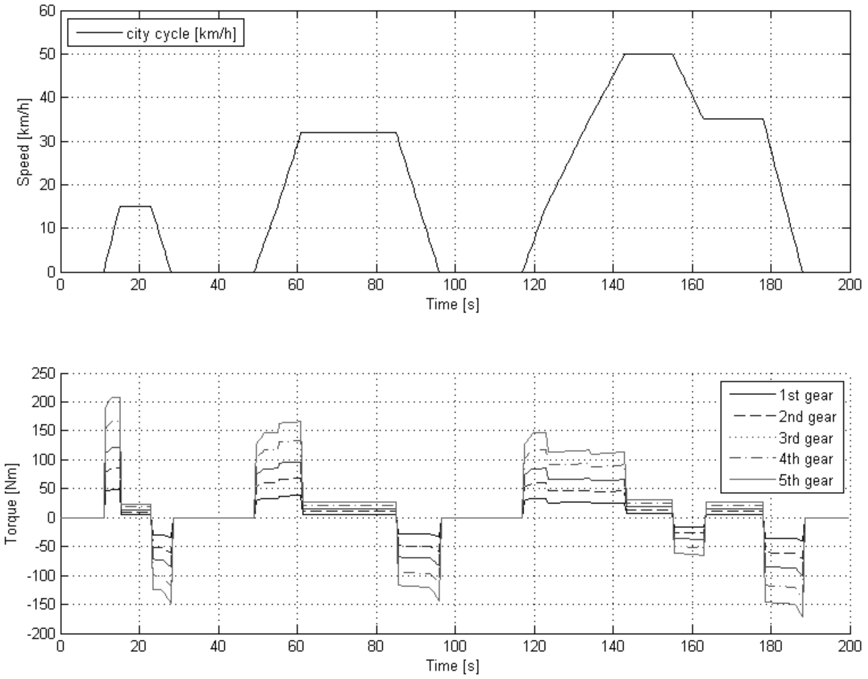

Fig 2-1 required engine torques

Electrical system

Electrical engine

The electrical system consists of the electrical motor and the battery. This combination makes it possible to transform mechanical energy into electrical energy and vice versa. The mechanical energy can be drawn from the combustion engine as well as the kinetic energy of the vehicle. For the electrical engine a characteristic map for the current has to be determined. To generate the map, a grid of torques and revolution speeds has to be established. The current for every point of that grid has to be measured. An example for such a map is shown in **Fig 2-2.** The current is measured at a defined constant voltage.

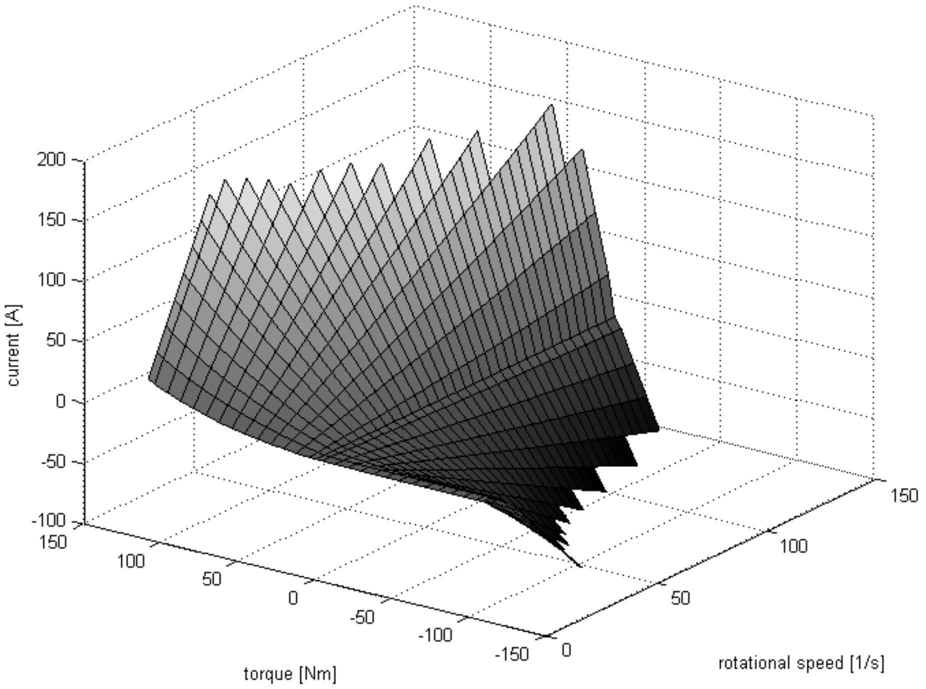

Fig 2-2 characteristic map of the electrical engine

As the battery voltage will change with battery State-of-Charge (as from now abbreviated SoC) and battery current, electrical power has to be calculated from the current map by multiplying with the operation voltage used during current measurement.

$$P_{em}(\tau,n)=I(\tau,n)\times U_{meas} \tag{2.6}$$

Using a degree of efficiency map is not appropriate for this purpose, as it depicts the relation between acquired mechanical power and electrical power. Regarding a start-up procedure, the mechanical power is zero as the angular speed is zero and so the product of angular speed and torque (mechanical power) is also zero. Still a current is required to produce this torque, making the degree of efficiency zero. This makes calculation of required current for a desired torque impossible, so a current map is needed.

Battery

The battery model is based on the Randles battery model (**Fig 2-3**). This model is an origin for a variety of more complex models [3] As the simulation/optimization method is based on quasi-static states, the inductive and capacitive components are neglected. Thus only the SoC and the internal resistance will be modelled.

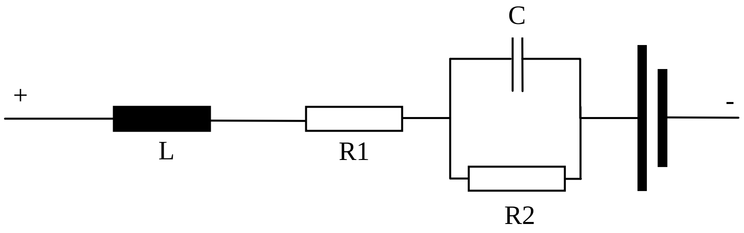

Fig 2-3 Randles battery model

To model the battery terminal voltage, the dependency of voltage and SoC has to be determined. The terminal voltage has to be determined for a given SoC and given current. The SoC influence has to be modelled by a characteristic curve.

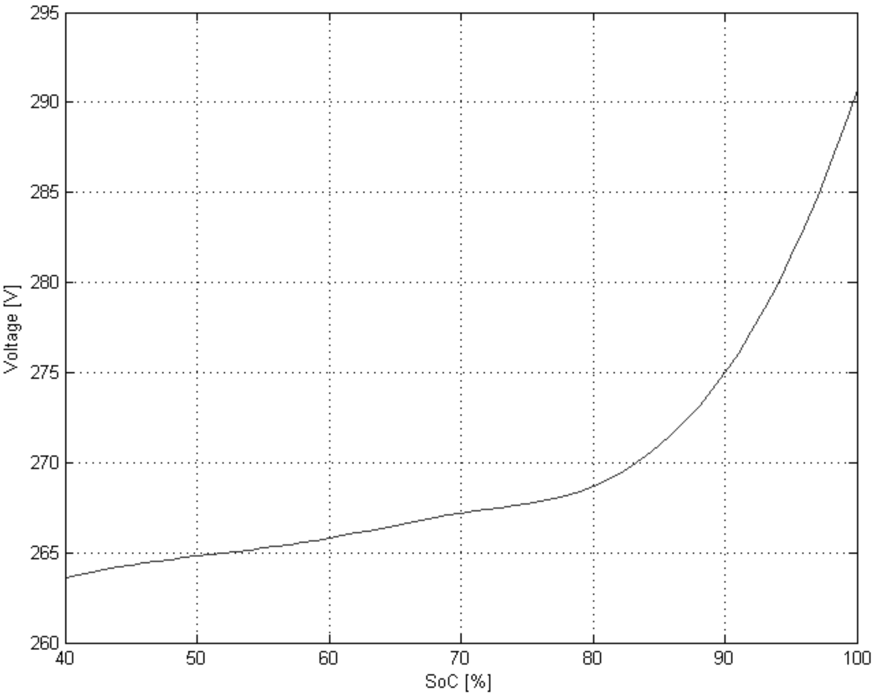

Fig 2-4 Battery voltage curve

The terminal voltage is also influenced by the current through the internal resistance.

$$U_{term} = U_i(SoC) + I \times R_i \tag{2.7}$$

U_{term} … terminal voltage

U_i … internal voltage (SoC-dependent)

I … current (positive values mean loading)

R_i … internal resistance

The State of Charge is the integration of the current flowing into the battery, so a constant factor similar to the capacity of a capacitor can be determined. Although secondary batteries (e.g. NiMH batteries) used in hybrid-vehicles naturally discharge with time, this effect can be neglected for optimization runs spanning well under an hour.

$$SoC(t) = C_{bat} \times \int_{t_0}^{t} I(t)dt + SoC_0 \tag{2.8}$$

SoC … State of Charge

SoC_0 … initial SoC

C_{bat} … equivalent battery capacity

Current calculation

To determine the SoC-change through equation (2.8) in dependance of the mechanical power of the electrical engine, the corresponding current has to be calculated.

$$I = \frac{P_{em}}{U_{term}} \tag{2.9}$$

With (2.6, 2.7):

$$I = \frac{P_{em}}{U_i + R_i \times I}$$

$$I = \frac{-U_i + \sqrt{U_i^{\,2} + 4P_{em}R_i}}{2R_i} \tag{2.10}$$

Combustion engine

For the determination of the carbon dioxide emissions of the combustion engine, an emissions map is needed.

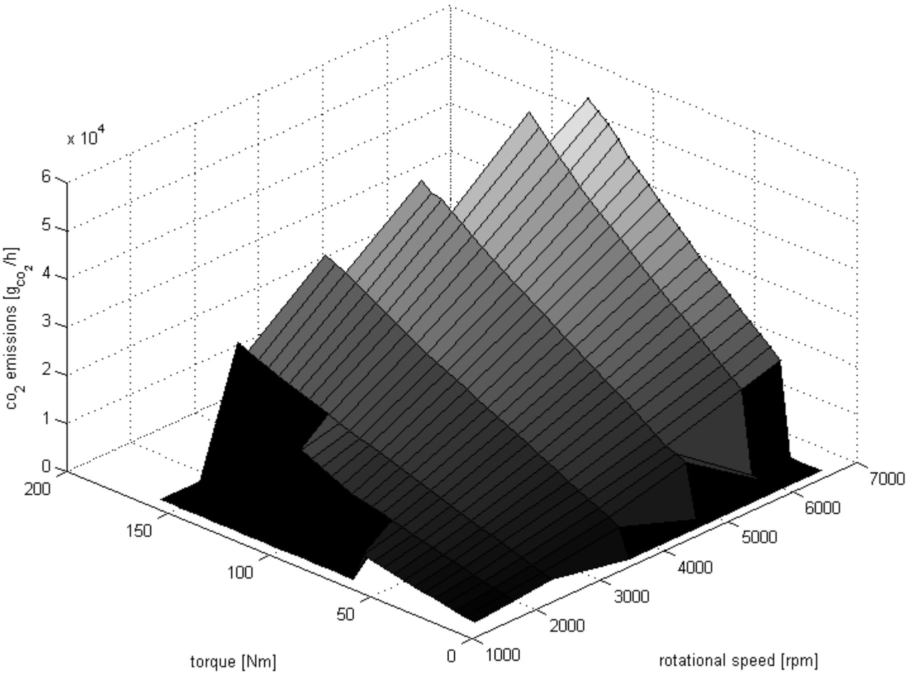

Fig 2-5 Exemplary CO_2 emissions for a combustion engine

3 Optimization

Goal of this research is to find the optimal way to drive a given road route in respect of CO_2 emissions with a parallel-hybrid vehicle. Degrees of freedom for that task are the chosen gear and division of needed driving torque to the two engines. An appropriate optimization method to solve this problem is dynamic programming. It is based on the Bellman-equation.

„An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.“ – *Bellman, 1957*

This means that for each part of the solution vectors, the „sub“-solution has to be the optimal-way between these two points.

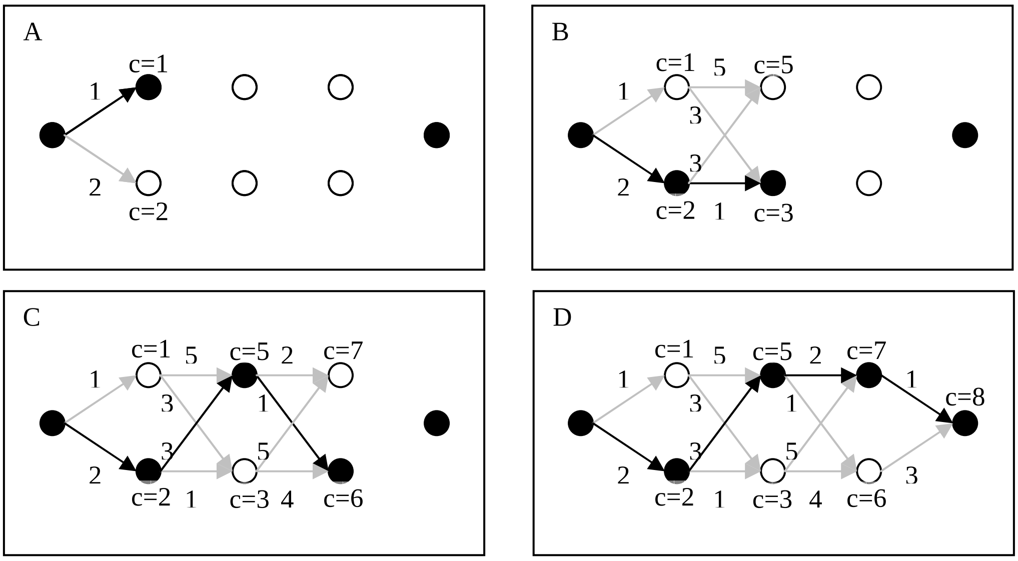

Fig 3-1 Dynamic Programming Step-by-step decision process

Fig 3-1 illustrates the step-by-step decision process. For every state of the current step, the „cheapest“ sum of any state in the step before and the corresponding way to the current state in the current step has to be found. As there is only one state in the final step, the sum of the cost for the transition to the final state plus the overall cost to the preceding states decide the overall cost. So for every state in the current step only as many decisions as there are preceding states have to be made. So for n steps and x states the number of necessary decision is:

$$d = x^2 \times (n-3) + 2 \times x \tag{3.1}$$

d … number of necessary decisions
x … number of states
n … number of steps

The overall number of possible combinations calculates to

$$c = x^{n-2} \tag{3.2}$$

c … number of combinations

So for an optimization problem consisting of 200 steps and 100 possible states, using dynamic programming leads to 1.97×10^6 decisions, compared to 10^{396} possible combinations.

4 Problem modelling

For comparability to production cars, the NEDC-cycle (New European driving cycle) was chosen (**Fig 4-1**).

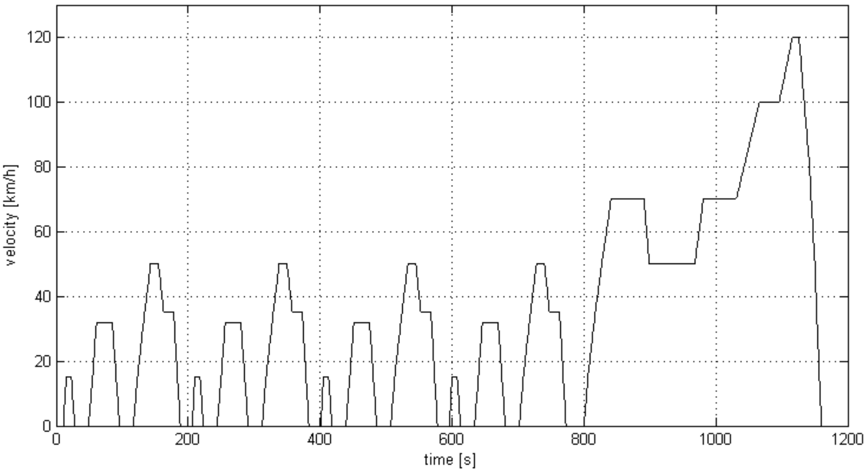

Fig 4-1 New European driving cycle

To find the optimal way to drive a given route, an internal state of the system has to be chosen. In this case, the SoC of the battery was chosen. So the desired SoC-range is divided into an appropriate number of SoC-levels. The number of levels chosen is determined by the current necessary to go one step up/down within one time step. The current to in/decrease the SoC-level by one discrete step can be determined by transforming (2.8):

$$I = \frac{\Delta SoC}{C_{bat} \times \Delta t} \tag{4.1}$$

ΔSoC … SoC-level unit interval
Δt … step width

The SoC- and time-resolution should be chosen in a way, that the current (and thus the electrical power at an average battery voltage level) necessary for a SoC-in/decrease by one step corresponds to a reasonable torque resolution in the torque range of the electrical/combustion engines even at lower rotational speeds. For this problem no general solution can be presented, as it depends on the regarded components. In the research this paper is based on, a step width of 0.5 s and a resolution of 1000 SoC-levels for 10 % SoC-range was chosen.

From the necessary driving torques (as calculated in Chapter 2), the driveable gears have to be determined. An important difference to the dynamic programming example shown in Chapter three is that the state transitions can be executed in different gears. So each transition has to be evaluated per driveable gear. This can be thought of as multivalued transitions. A gear is driveable if the sum of the maximum torques of both engines is higher than the required driving torque in that gear for positive (powering) torques. For negative torques (decelerating), all gears are possible as not all the torque has to be produced by the electrical engine. As the brake system of the car remains unmodified, only a part of the braking torque can be recuperated by the electrical system, so an upper torque percentage limit has to be defined. For the car this work is based on, the maximum torque regarded throughout the optimization procedure for the electrical engine equals a third of the decelerating torque needed.

To allow battery loading during driving breaks (e.g. stops at traffic lights), an engine rotational speed is defined. It was set to 2500 rpm. This speed was chosen as the torque range is at its maximum and the area of optimal degree of efficiency is also located at that rotational speed. The minimum torque is set to zero, the maximum torque is the lower of the maximum torques of both engines. The required driving torque in this case is zero.

Knowing the driveable gears, the maximum and minimum of the torque applicable for the electrical engine has to be determined. The maximum torque is the minimum selection of the torque the electrical engine can provide and the torque that is needed in the selected gear. The minimum torque is the difference between the required driving torque and the maximum torque the combustion engine can provide.

$$\tau_{dr}(i,t) - \tau_{\max\_ce}(n) < \tau_{em}(t) < \min(\tau_{\max\_em}(n), \tau_{dr}(i,t)) \qquad (4.2)$$

τ_{dr} … driving torque

$\tau_{\max\_ce}$ … maximum torque of combustion engine

$\tau_{\max\_em}$ … maximum torque of electrical engine

i … gear

From the possible torque range and the rotational speed in the regarded gear, the electrical current limits can be calculated by (2.10). By solving (4.1) for ΔSoC, the possible amount of SoC in/decrease steps for this time step and the regarded gear can be determined. As not all transitions between states of adjacent steps are possible because of the limited SoC in/decrease range, the theoretical complexity calculated with (3.1) is by far beyond the necessary number of calculations. Thus the computation time for this method is acceptable for theoretical research.

With the possible SoC-level step range, the necessary electrical engine torques for the exact current to make the desired SoC-level steps have to be determined by reversing the characteristic map for the electrical engine (Fig 2-2) to provide torque from given rotational speed and current.

The difference between the necessary driving torque and the electrical engine torque equals the necessary combustion engine torque. By looking up the emitted CO_2 for the current rotational speed and the determined necessary torque fraction for the combustion engine in Fig 2-5, the

„cost“ for the regarded route leading to this specific state (SoC-level) can be calculated by adding the so-far emissions to the origin state of the current consideration. If no better state vector had been found, the state and gear vector as well as the CO_2 emissions are stored for the regarded destination state. For simplicity reasons, the electrical engine torque vector is also stored, although it could be reconstructed from the SoC-level and gear vectors after the optimization is done.

5 Results

After the optimization run, the following results were derived. Fig 6-1 shows an excerpt of the progression of the regarded vectors. Shown are the required torque in the gear selected during the optimization run and the optimal electrical engine torque vector, the acquired gear and the SoC progression.

The results feature multiple kinds of hybrid operating modes. During braking manoeuvres, kinetic energy is regained by recuperation. At constant velocity, the combustion engine operating point is shifted to higher loads by producing negative torque. This way the battery is charged. The stored energy is used to support the combustion engine, making it possible to select a higher gear.

It is obvious that the result shows the discharge of the battery just before it is recharged at the long deceleration manoeuvre at the end of the cycle. Recuperation is always done when possible.

The CO_2 emissions determined for the used middle-class caravan were about 88.27 g CO_2/km. Compared to the production car (1.6 liter gasoline engine, no hybrid), emitting approximately 150 g CO_2/km, the potential is significant. Still there would be compromises to be made in respect of driveability, comfort and comprehensibility, so a production car featuring the components simulated in this research would emit more CO_2/km than the calculated value.

6 Conclusion

The chosen optimization approach is well suited for the given problem. It is a good way to determine the potentials for a given hardware configuration. It can also help in developing an appropriate strategy of operation and to evaluate an already established solution. Further research is required to fully understand the determined optimal solution.

For a real-time implementation of the described method, the computation times are problematic. Depending on the valid SoC-range, SoC-resolution and the regarded horizon, real-time calculation can become impossible [5] . An operation strategy for a vehicle with a high-capacity battery, making it possible to use electric drive for longer periods, has to regard a far optimization horizon. Using neural networks, trained to deliver the results, the offline optimization would have, could be a solution to this problem.

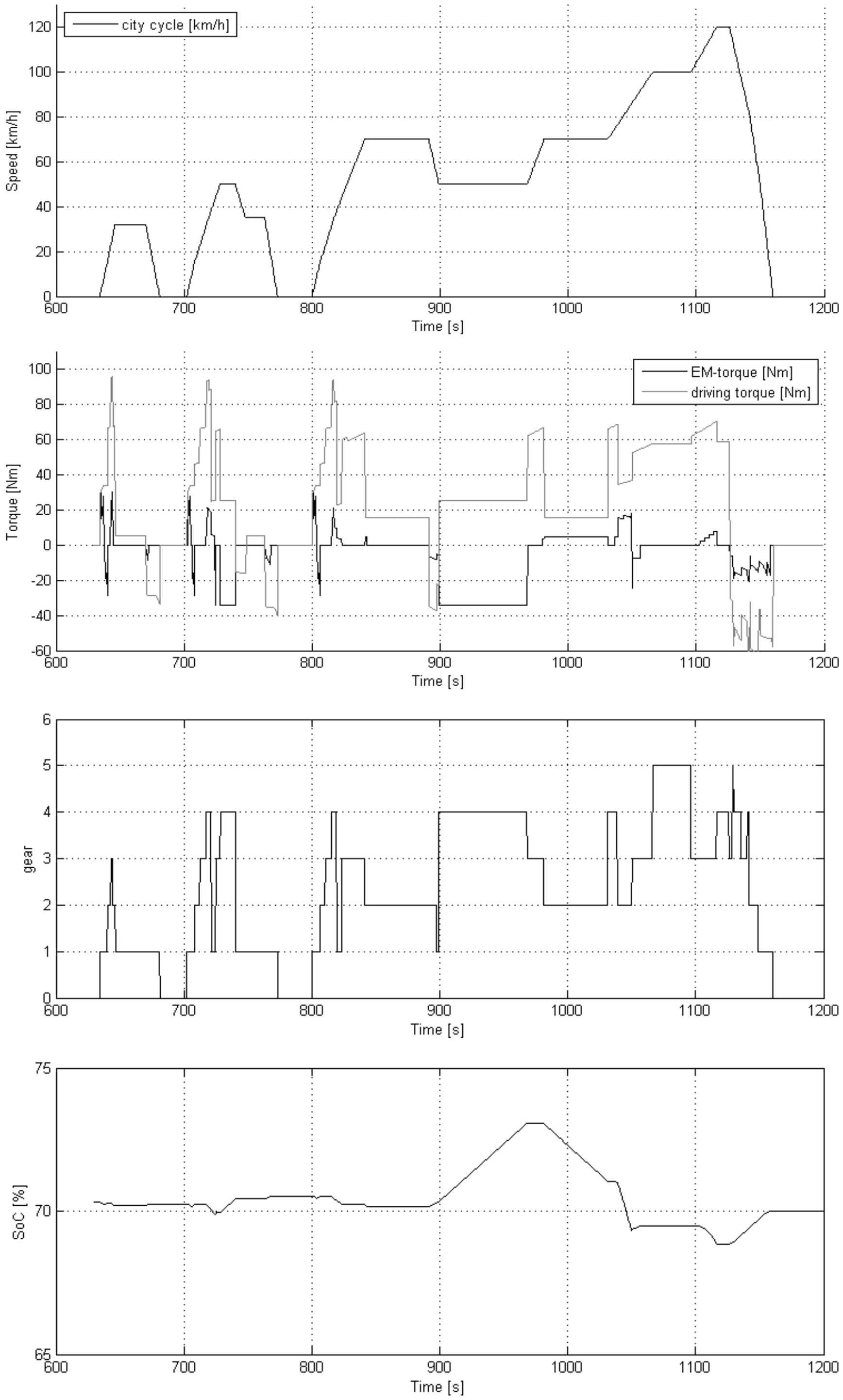

Fig 6-1 Optimization results, electrical engine torque, gear & SoC

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